GUT and Supersymmetry

Hitoshi Murayama

129A

F2002 semester

Grand Unified Theories

Motivations for GUT

- Charge quantization, anomaly cancellation, bizarre hypercharge assignments in the Standard Model
- Three seemingly unrelated forces yet all gauge forces
- Einstein's dream towards a unified description of all forces
- Baryogenesis no longer a prime motivation

Quantum Numbers in the Standard Model

- I didn't become a physicist to memorize these weird numbers...

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,-\tfrac{1}{6}) \quad u_R(3,1,+\tfrac{2}{3}) \quad d_R(3,1,-\tfrac{1}{3})$$

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \quad l_R(1,1,-1)$$

Quantum Numbers in the Standard Model

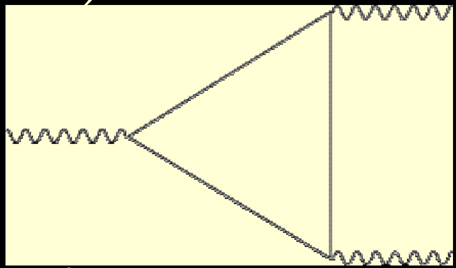
- To treat them on equal footing, make all particles left-handed using CP

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,+\tfrac{1}{6}) \qquad \bar{u}_L\,(3^*,1,-\tfrac{2}{3}) \qquad \bar{d}_L\,(3^*,1,\tfrac{1}{3})$$

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \qquad \bar{l}_L\,(1,1,1)$$

Gauge Anomaly

- Gauge symmetry crucial to keep quantum field theories (including the SM) under control
- Triangle diagrams:



- May spoil the gauge invariance at quantum level → disaster
- Anomalies must all vanish for three gauge vertices (not for global currents, e.g. *B*, *L*)
- Sum up all standard model fermions and see if they indeed vanish

Anomaly Cancellation

- $U(1)^3$ $\quad 3\cdot 2\left(\tfrac{1}{6}\right)^3 + 3\left(\square\tfrac{2}{3}\right)^3 + 3\left(\tfrac{1}{3}\right)^3 + 2\left(\square\tfrac{1}{2}\right)^3 + (1)^3 = 0$
- $U(1)(\text{gravity})^2$ $\quad 3\cdot 2\left(\tfrac{1}{6}\right) + 3\left(\square\tfrac{2}{3}\right) + 3\left(\tfrac{1}{3}\right) + 2\left(\square\tfrac{1}{2}\right) + (1) = 0$
- $U(1)(SU(2))^2$ $\quad 3\cdot 2\left(\tfrac{1}{6}\right) + 2\left(\square\tfrac{1}{2}\right) = 0$
- $U(1)(SU(3))^2$ $\quad 3\cdot 2\left(\tfrac{1}{6}\right) + 3\left(\square\tfrac{2}{3}\right) + 3\left(\tfrac{1}{3}\right) = 0$
- $(SU(3))^3$ $\quad \#\underline{3}\square\#\underline{3}^* = 2\square 1\square 1 = 0$
- $(SU(2))^3$, $(SU(3))^2 SU(2)$, $SU(3)(SU(2))^2$ $\quad 0$
- $SU(2)$ $\quad \#\underline{2} = 3 + 1 = 4 = \text{even}$

Non-trivial connection between *q* & *l*

SU(5) GUT

- SU(3)$\times$SU(2)$\times$U(1)$\subset$SU(5)
- U(1) must be traceless: try 5*: $\begin{pmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ \nu \\ l \end{pmatrix}$
- 5$\times$5 matrices

$$\mathrm{SU(3)} \begin{pmatrix} -\tfrac{1}{2}\lambda^{a*} & 0 \\ 0 & 0 \end{pmatrix} \quad \mathrm{SU(2)} \begin{pmatrix} 0 & 0 \\ 0 & \tfrac{1}{2}\sigma^{a} \end{pmatrix}$$

$$\mathrm{U(1)} \begin{pmatrix} \tfrac{1}{3} I_3 & 0 \\ 0 & -\tfrac{1}{2} I_2 \end{pmatrix}$$

SU(5) GUT

- Then the rest belongs to <u>10</u>
- All quantum numbers work out this way

$$\begin{pmatrix} 0 & \bar u & -\bar u & d & -u \\ -\bar u & 0 & \bar u & d & -u \\ \bar u & -\bar u & 0 & d & -u \\ -d & -d & -d & 0 & \bar l \\ u & u & u & -\bar l & 0 \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,-\tfrac{1}{6}) \sim \left[\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \quad \bar d_L (3^*,1,\tfrac{1}{3}) \right]^*$$

$$\bar u_L (3^*,1,-\tfrac{2}{3}) \sim \left[\bar d_L (3^*,1,\tfrac{1}{3}) \quad \bar d_L (3^*,1,\tfrac{1}{3}) \right]^*$$

$$\bar l_L (1,1,1) \sim \left[\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \quad \begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \right]^*$$

- Anomaly cancellation: $\#\underline{10} - \#\underline{5^*} = 0$

Fermion Mass Relation

- Down- and lepton-Yukawa couplings come from the same SU(5) operator 10 5* H
- Fermion mass relation

$$m_b = m_\tau,\ m_s = m_\mu,\ m_d = m_e$$

- Reality:

$$m_b = m_\tau,\ 3m_s = m_\mu,\ m_d = 3m_e$$

- Not bad!

SO(10) GUT

- $SU(5) \times U(1) \subset SO(10)$

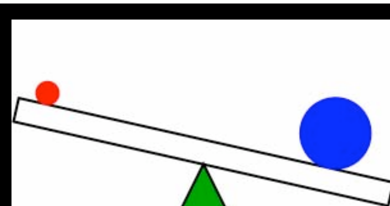
  $$16 = (10, +1) + (5^*, -3) + (1, +5)$$

- Come with right-handed neutrinos!
  - anomaly-free for any multiplets
  - Smallest simple anomaly-free group with chiral fermions
  - Smallest chiral representation contains all standard model fermions

Seesaw Mechanism

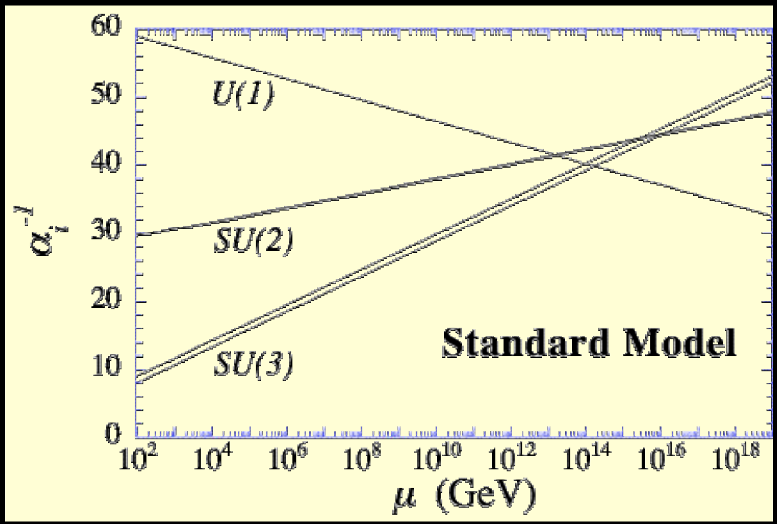
- Once SO(10) broken to the standard model, right-handed neutrino mass becomes allowed by the gauge invariance $M \sim h\, M_{\mathrm{GUT}}$

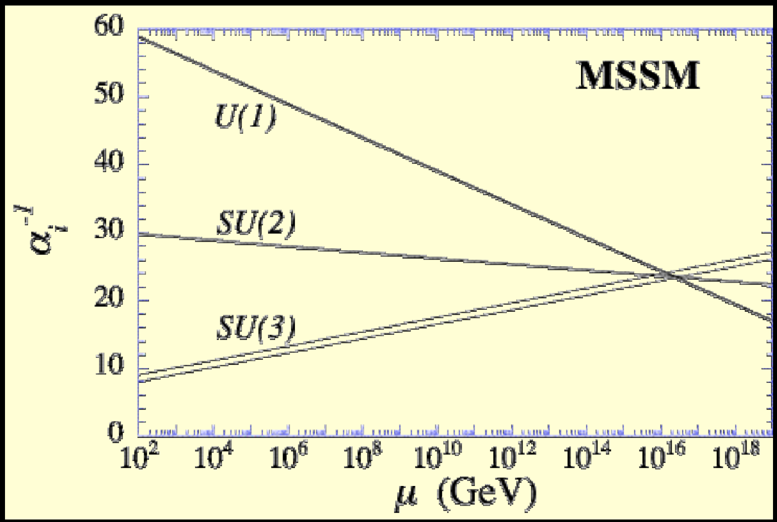
$$(\nu_L \quad \nu_R)\begin{pmatrix} & m_D \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \qquad m_\nu = \frac{m_D^2}{M} << m_D$$



To obtain $m_3 \sim (\Delta m^2{}_{\mathrm{atm}})^{1/2}$, $m_D \sim m_t$, $M_3 \sim 10^{15}$GeV (GUT!)

Gauge Coupling Unification

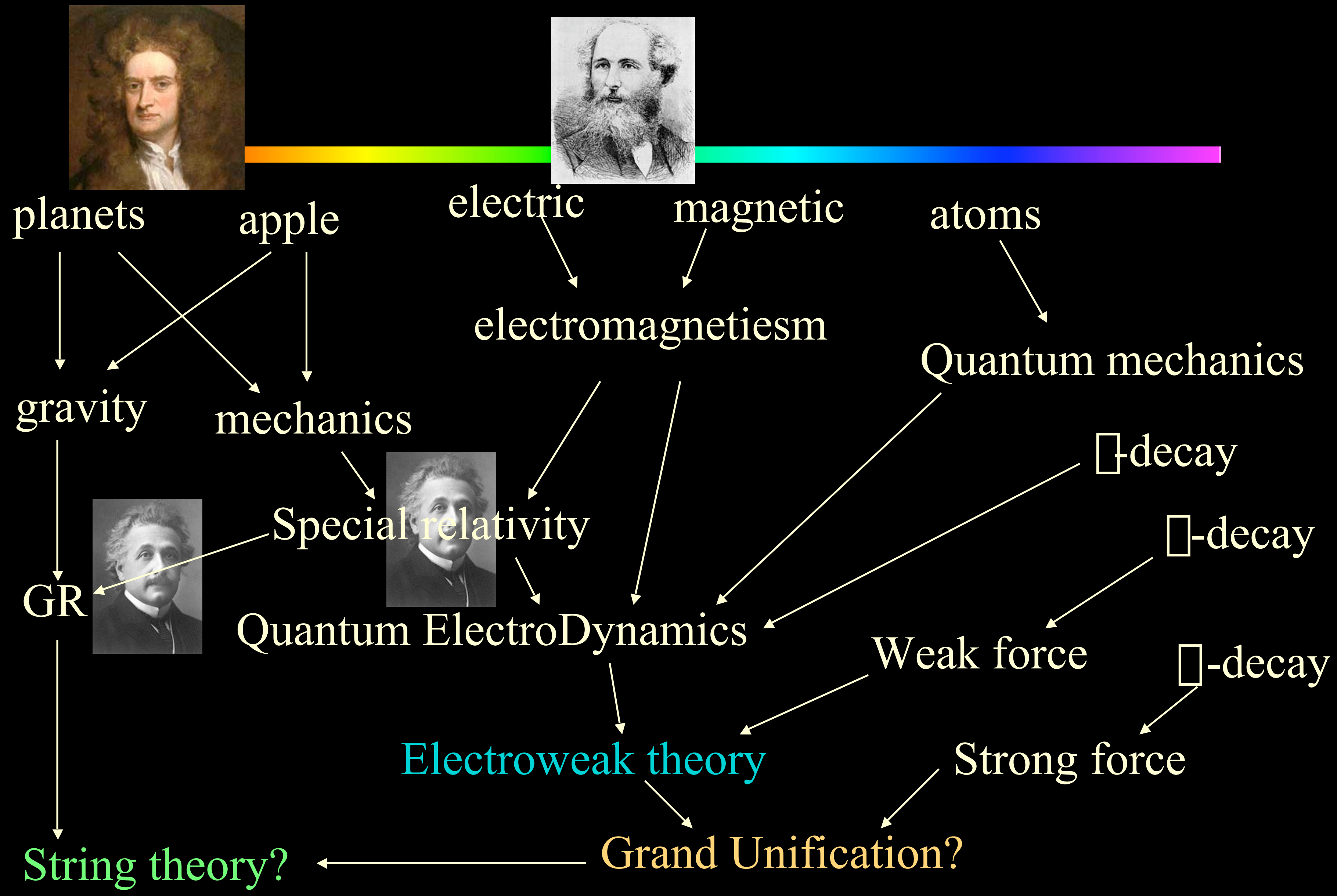




Einstein's Dream

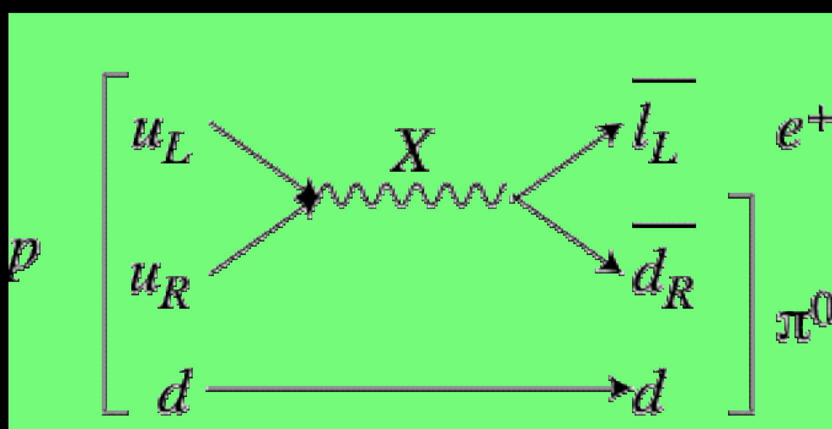
- Is there an underlying simplicity behind vast phenomena in Nature?
- Einstein dreamed to come up with a *unified* description
- But he failed to unify electromagnetism and gravity (GR)

History of Unification



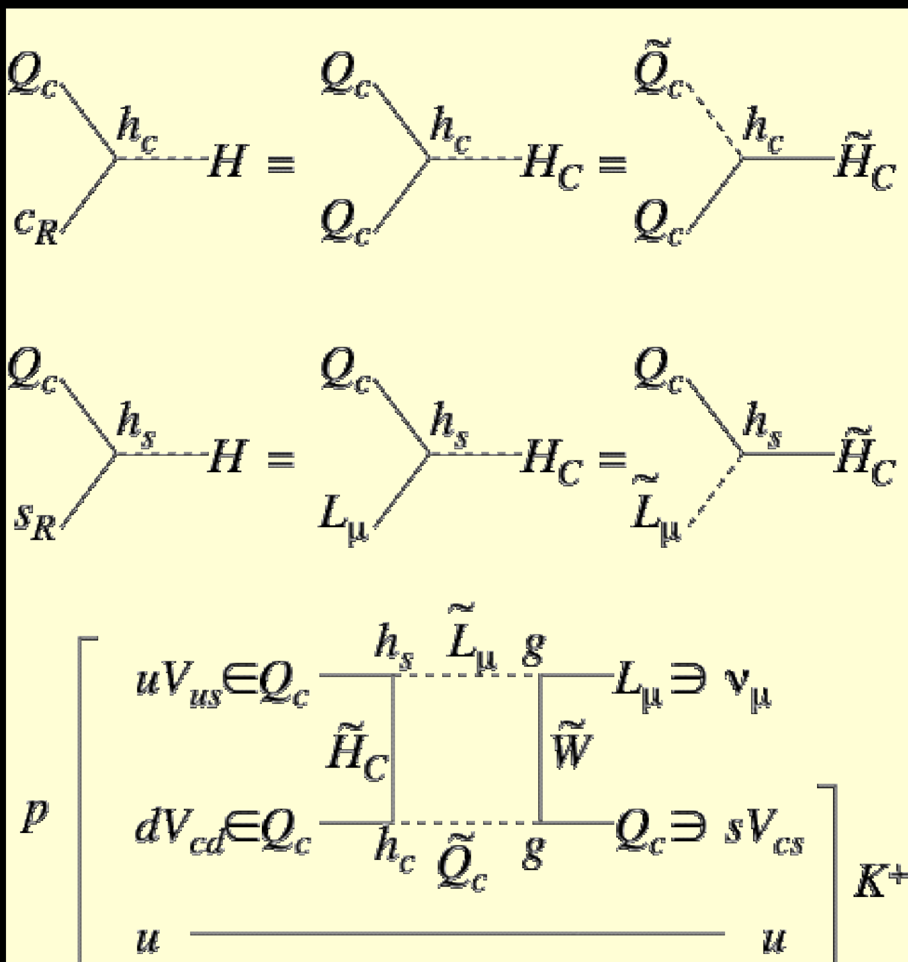
Proton Decay

- Quarks and leptons in the same multiplet
- Gauge bosons can convert *q* to *l*
- Cause proton decay!



$$\sim \left(\frac{g^2}{M_X^2}\right)^2 m_p^5$$

Supersymmetric Proton Decay

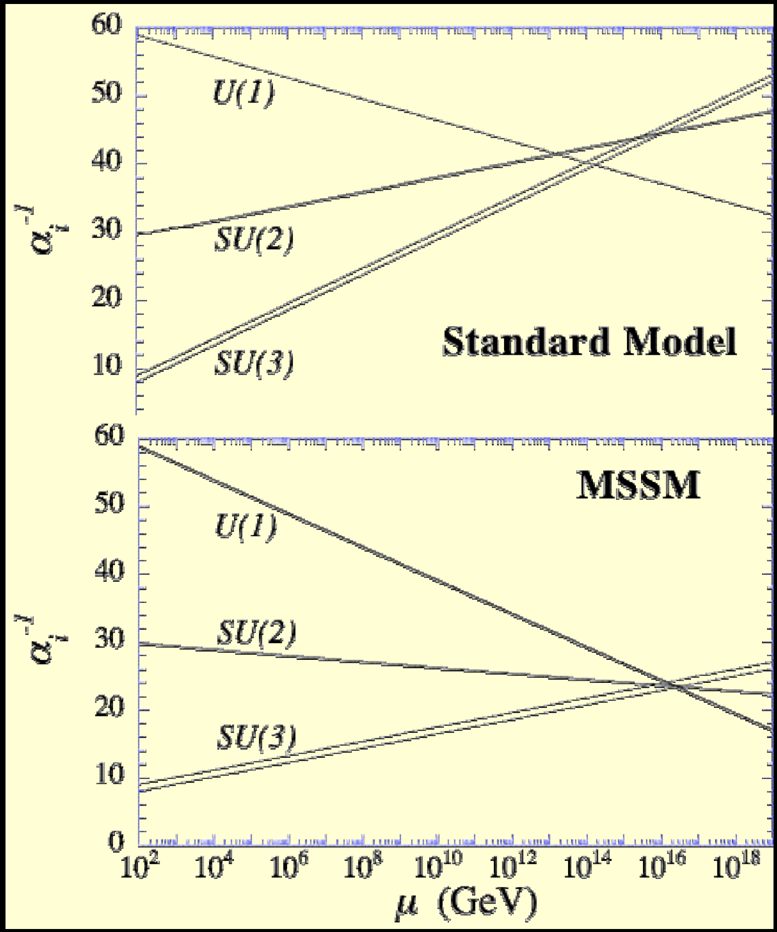


$$\sim \left(\frac{g^2}{(4\pi)^2} \frac{h_s h_c \theta_C^2}{M_{H_C} m_{SUSY}} \right)^2 m_p^5$$

Suppressed only by the *second power* of GUT scale vs *fourth* in *X*-boson exchange

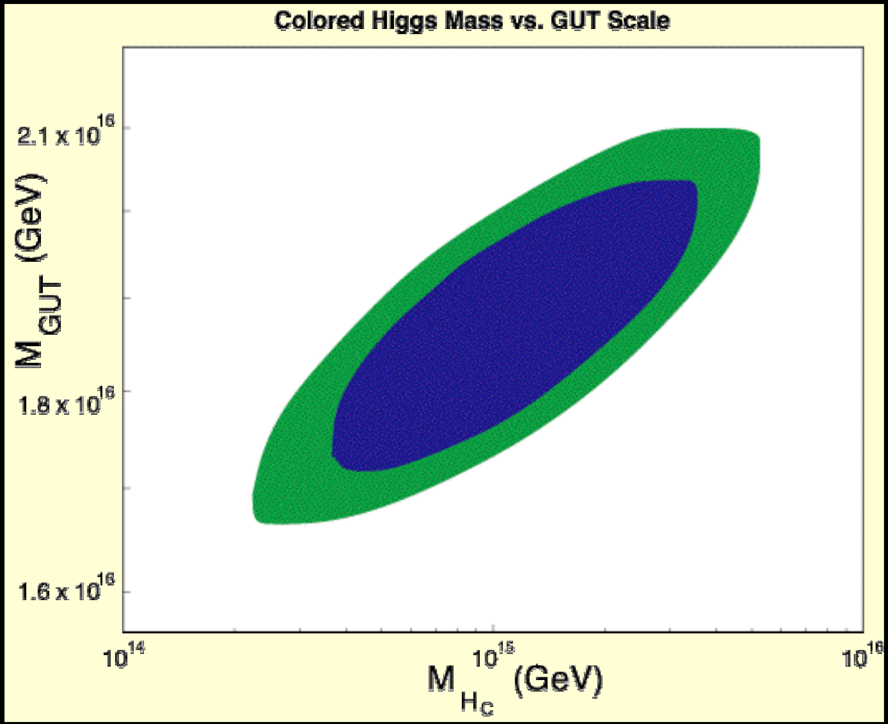
Proton Decay

- No sign of proton decay yet!
  - Non-SUSY GUT does not unify couplings
- Minimal SUSY particle content
  - Couplings unify!
  - $\tau(p \to K^+ \bar{\nu}) > 6.7\ 10^{32}$ years (90% CL) from SuperK



Rest In Peace Minimal SUSY SU(5) GUT

- RGE analysis



- SuperK limit

  $M_{Hc} > 7.6\ 10^{16}$ GeV

- Even if 1st, 2nd generation scalars "decoupled", 3rd generation contribution (Goto, Nihei)

  $M_{Hc} > 5.7\ 10^{16}$ GeV

  (HM, Pierce)

Avoiding Proton Decay

- Unfortunately, proton decay rate/mode is highly model-dependent
  - more threshold corrections (HM, Pierce)
  - Some fine-tuning (Babu, Barr)
  - GUT breaking by orbifolds (Kawamura; Hall, Nomura)
  - Depends on the triplet-doublet splitting mechanism, Yukawa (non-)unification

Don't give up!

- Still, proton decay unique window to physics at $>10^{15}$ GeV
- Suppression by fine-tuning: $p \to K^+\bar{\nu}$ may be just around the corner
- Flipped SU(5): $p \to e^+\pi^0$ possible
- We still need SuperK!
- Eventually with ~1000kt detector

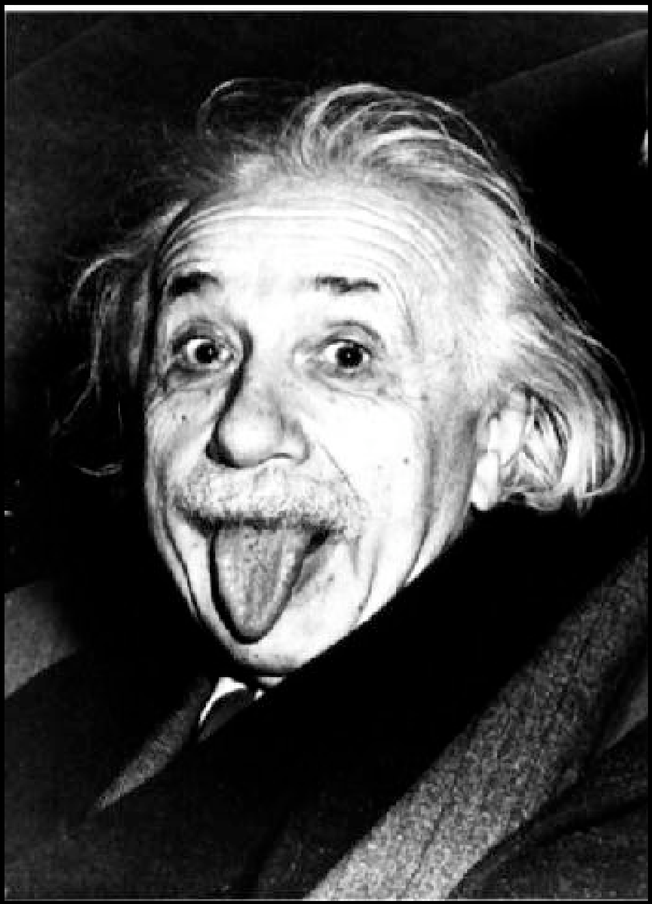
Supersymmetry

Why was Anti-Matter Needed?

- At the end of 19th century: a “crisis” about electron
  - Like charges repel: hard to keep electric charge in a small pack
  - Electron is point-like
  - At least smaller than 10^{-17} cm
- Need a lot of energy to keep it small!

$E=mc^2$

- Need more than 10^9 eV of energy to pack electric charge tightly inside the electron
- But the observed energy of the electron is only 5 10^5 eV
- Electron cannot be smaller than 10^{-13} cm??
- Breakdown of theory of electromagnetism

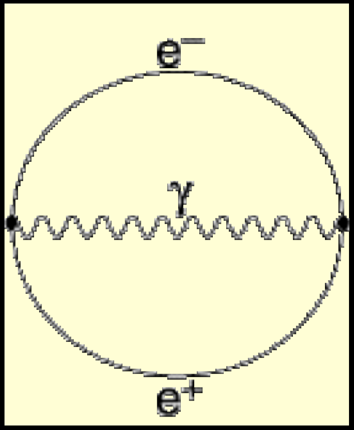


Uncertainty Principle

- Energy-Time Uncertainty Principle:

  You can violate energy conservation if it is only for a short time

- Vacuum is full of quantum bubbles!

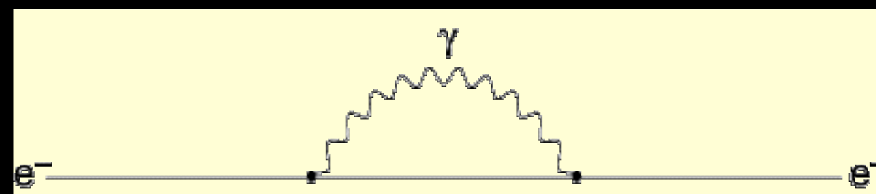


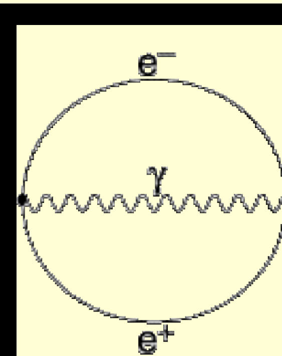
Werner Heisenberg

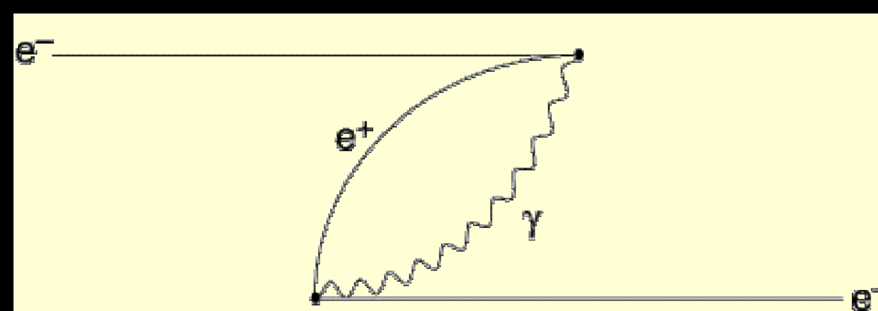
Anti-Matter Helps

- Electron creates a force to repel itself
- Vacuum bubble of matter anti-matter creation/annihilation
- Electron annihilates the positron in the bubble

   only 10% of mass





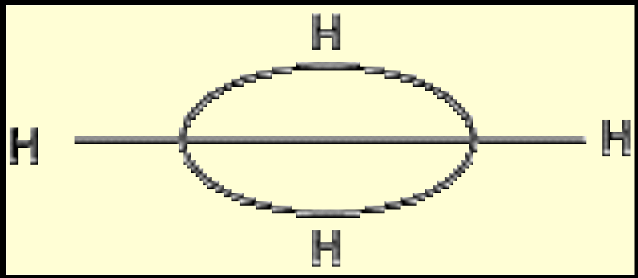


Anti-Matter Helps

- "Anti-matter attraction" cancels "Like-charge repulsion"
- It does not cost too much energy to tightly pack the electric charge inside the electron
- Needed anti-matter: double #particles
- Theory of electromagnetism now works at very short distances (12 digits accuracy!)

Higgs repels itself, too

- Just like electron repeling itself because of its charge, Higgs boson also repels itself
- Requires a lot of energy to contain itself in its point-like size!
- Breakdown of theory of weak force

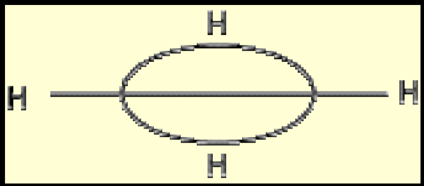


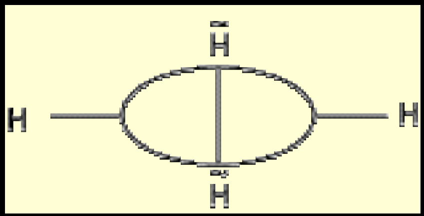
But there is gravity

- Gravity and quantum mechanics unify at an extremely short distance 10^{-33} cm
- Higgs boson must be this small, too, to have a sensible unified theory of gravity and quantum mechanics
- But current theory of weak force breaks down already at 10^{-17} cm

History repeats itself?

- Double #particles again → superpartners
- “Vacuum bubbles” of superpartners cancels the energy required to contain Higgs boson in itself
- Theory of weak force made consistent with unification of gravity and quantum mechanics



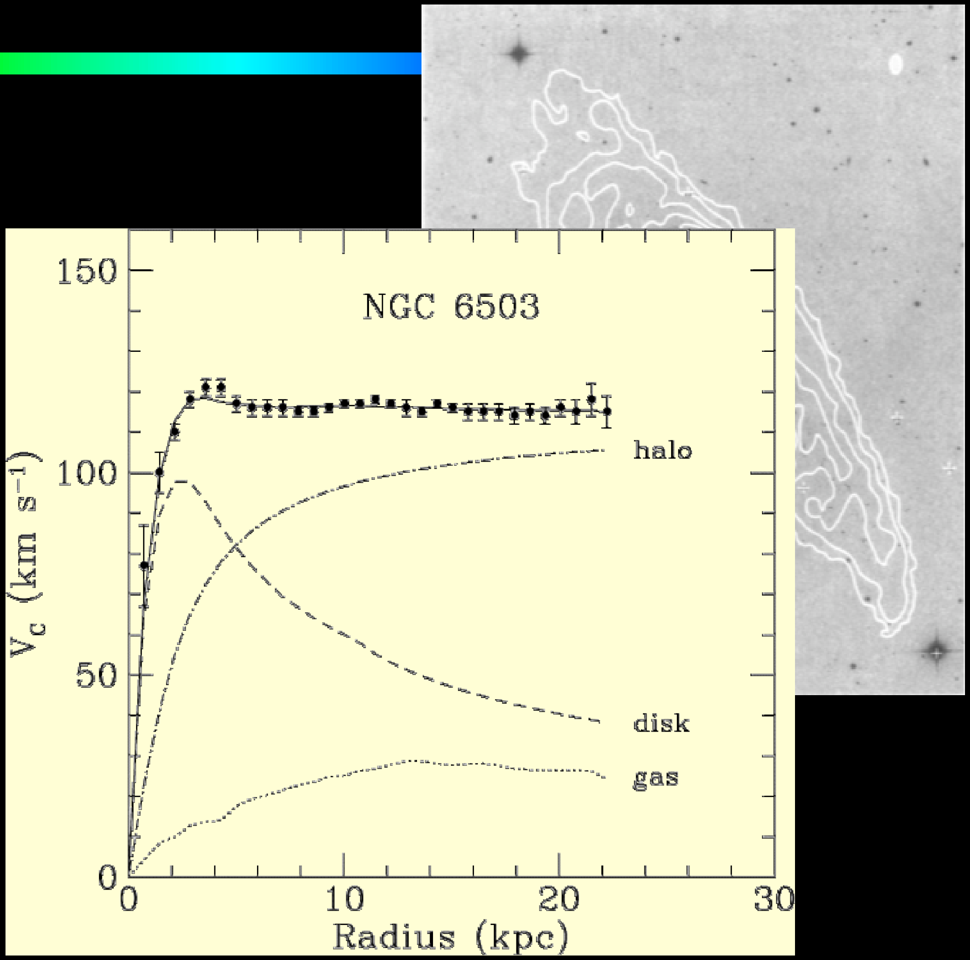


Where are the superpartners?

- They need to cancel self-repelling energy of the Higgs boson
- Cannot be too heavy to do this job
- Have to be below 10^{12} eV or “Fermi energy”
- We are getting there this decade
  - Tevatron (Fermilab, Illinois) 2001–
  - LHC (CERN, Switzerland) 2006–

Superpartners everywhere?

- There are unknown "Dark Matter" in our galaxy and outside
- It amounts for about 30% of the Universe
- Lightest superpartner one of the best candidates



Superpartners as probe

- Most exciting thing about superpartners beyond existence:

  They carry information of small distance physics to something we can measure

  e.g., *"Is Grand Unification true?"*