HW #8 (221A), due Nov 3, 4pm

1. Consider the Stern–Gerlach experiment for spin 1. When the atom enters with $J_z = +\hbar$ in the magnetic field along the x axis, determine the relative strengths of three lines that correspond to $J_x = +\hbar, 0, -\hbar$.

2. The quadrupole moment operators can be arranged into spherical tensors operators

$$Q^{(+2)} = \sqrt{\frac{3}{8}}(x+iy)^2 \qquad (1)$$

$$Q^{(+1)} = -\sqrt{\frac{3}{2}}(x+iy)z \qquad (2)$$

$$Q^{(0)} = \frac{1}{2}(3z^2 - r^2) \qquad (3)$$

$$Q^{(-1)} = \sqrt{\frac{3}{2}}(x-iy)z \qquad (4)$$

$$Q^{(-2)} = \sqrt{\frac{3}{8}}(x-iy)^2 \qquad (5)$$

   Using the form of the wave function $\psi_{lm} = R(r)Y_l^m(\theta,\phi)$,

   (a) Calculate $\langle\psi_{3,3}|Q^{(0)}|\psi_{3,3}\rangle$.

   (b) Predict all others $\langle\psi_{3,m'}|Q^{(k)}|\psi_{3,m}\rangle$ using Wigner–Eckart theorem in terms of Clebsch–Gordan coefficients.

   (c) Verify them with explicit calcuations for $\langle\psi_{3,1}|Q^{(1)}|\psi_{3,0}\rangle$, $\langle\psi_{3,-1}|Q^{(-2)}|\psi_{3,1}\rangle$, and $\langle\psi_{3,-2}|Q^{(0)}|\psi_{3,-3}\rangle$.

   Note that we leave $\langle r^2\rangle = \int_0^\infty r^2 dr R(r)^2 r^2$ as an overall constant that drops out from the ratios.

3. (optional) As it was done in the class, add angular momenta $j_1 = 3/2$ and $j_2 = 1$ and work out all Clebsch–Gordan coefficients starting from the state $|j,m\rangle = |\frac{5}{2},\frac{5}{2}\rangle = |\frac{3}{2},\frac{3}{2}\rangle \otimes |1,1\rangle$.

4. (optional) Answer following questions about the spherical harmonics.

   (a) Show that L_+ annihilates $Y_2^2 = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\phi}$.

(b) Work out all of Y_2^m using successive applications of L_- on Y_2^2.

(c) Plot the "shapes" of all Y_2^m as explained in the lecture notes and shown in a sample Mathematica notebook.