HW #8 (221B), due Apr 1, 4pm

1. Consider a one-dimensional problem of two heavy particles at x_1 and x_2 of mass M and one light particle at x_3 of mass m attached by springs, with the light particle in the middle: $x_1 < x_3 < x_2$. The Hamiltonian of the system is

$$H = \frac{p_1^2}{2M} + \frac{p_2^2}{2M} + \frac{p_3^2}{2m} + \frac{1}{2}k(x_3 - x_1 - d)^2 + \frac{1}{2}k(x_2 - x_3 - d)^2. \qquad (1)$$

Here, d is the natural length of the spring and k the spring constant. Answer the following questions.

   (a) Use Bohr–Oppenheimer approximation to study the system. First, fix the positions of heavy particles x_1 and x_2, and find the energy eigenvalues in the $M \to \infty$ limit. Second, consider the energy eigenvalues (as a function of x_1 and x_2) as a potential energy for the heavy particles and work out energy eigenvalues for the heavy particles.

   (b) One can also solve this system exactly. Identify the basic oscillation modes, rewrite the Hamiltonian in terms of them, and obtain the energy eigenvalues exactly.

   (c) Compare two results and see what corrections are missed in Bohr–Oppenheimer approximation.

2. Identify low-lying levels in ^{14}C, ^{14}N, and ^{14}O that correspond to $I = 0$ and $I = 1$ multiplets. Use `http://ie.lbl.gov/TOI2003/LadderSearch.asp` for energy levels.