HW #9 (221B), due Apr 8, 4pm

1. Lookup levels of ^{210}Pb, ^{210}Bi, and ^{210}Po, and explain the low-lying excitations using the shell model.

2. Lookup levels of ^{17}O, and ^{17}F, and explain the low-lying excitations using the shell model.

3. Consider $2s$ and $2p$ wave functions of the hydrogen atom. Answer the following questions. Try to use Mathematica for visualization as much as possible.

   (a) Show that one can take linear combinations of states $|2p, m = \pm 1\rangle$ to obtain orbitals elongated along x or y axis (or equivalently, $\phi = 0$ and $\phi = \pi/2$ directions).

   (b) Show that one can further combine $|2s\rangle$ state to obtain orbitals elongated along $\phi = 0$, $\phi = 2\pi/3$, and $\phi = 4\pi/3$ directions. They are sp^2 mixed orbitals used for carbon atoms in ethylene, etc.

   (c) Similarly, show that one can combine all three $2p$ states and $2p$ state to obtain orbitals elongated along the axes of a tetrahedron. They are sp^3 mixed orbitals used in methane etc.

4. Consider the Lagrangian of a harmonic oscillator

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2. \tag{1}$$

   (a) Rewrite the Lagrangian using $p = \partial L/\partial \dot{x}$ and eliminate $\dot{x}$.

   (b) Further rewrite it using $a = \sqrt{\frac{m\omega}{2\hbar}}\, x + i\frac{1}{\sqrt{2m\hbar\omega}}\, p$ and $a^\dagger$.

   (c) Regarding a as the canonical coordinate, and comparing the Lagrangian to the general form $p\dot{q} - H(p, q)$, observe that the conjugate momentum is $i\hbar a^\dagger$ and derive the canonical commutation relation for $[a, a^\dagger]$ as well as the Hamiltonian in quantum mechanics.

   (d) Instead of using the usual canonical commutation relation, use an *anti*-commutation relation $\{a, a^\dagger\} = 1$, $\{a, a\} = \{a^\dagger, a^\dagger\} = 0$. Show that the Hilbert space consists of only two states, $|0\rangle$ and $|1\rangle = a^\dagger|0\rangle$. What are their energy eigenvalues?