HW #6, due Mar 3

1. DGLAP equations Study the evolution equations of $q(x, Q^2)$, $\bar{q}(x, Q^2)$, and $g(x, Q^2)$ using the DGLAP kernels given on the next page.

(a) If you keep only P_{qq} term in the kernel, verify that the "total number of the parton q" $\int_0^1 dx q(x)$ remains Q^2 independent only if there is the delta function piece in the kernel.

(b) The momentum conservation requires

$$\int_0^1 dx x \left(\sum_i q_i(x, Q^2) + \sum_i \bar{q}_i(x, Q^2) + g(x, Q^2) \right) = 1. \qquad (1)$$

Show that the delta function piece in P_{gg} is necessary to guarantee the above constraint to be satisfied for all Q^2.

2. Scalar Partons? If the partons have spin 0 instead of 1/2 (quarks), we obtain a different formula for the deep inelastic scattering. Following HW #5, and using the Feynman rules for the scalar QED (Peskin–Schroeder, p. 312), write down the cross section $d^2\sigma/dxdy$ in terms of parton distribution functions. The difference appears in the y-dependence.

DGLAP kernels

Notation: The DGLAP kernels $P_{ij}(z)$ refer to the splitting of the parton j into i and something else. The evolution of the parton distribution function for the parton species i is therefore given by

$$\frac{df_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{ij}(z) f_j(y,Q^2), \tag{2}$$

where $x = zy$. A summation over j is implied.

The kernels are given at the lowest order in QCD by

$$\begin{aligned}
P_{qq}(z) &= \frac{4}{3}\frac{1+z^2}{(1-z)_+} + 2\delta(1-z), && (3)\\
P_{gg}(z) &= 6\left(\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z)\right) + \left(\frac{11}{2} - \frac{n_f}{3}\right)\delta(1-z) \\
&= P_{gg}(1-z), && (4)\\
P_{gq}(z) &= P_{qq}(1-z), && (5)\\
P_{qg}(z) &= \frac{1}{2}(z^2 + (1-z)^2) \\
&= P_{qg}(1-z) && (6)\\
P_{\bar{q}g}(z) &= P_{qg}(1-z) && (7)\\
P_{g\bar{q}}(z) &= P_{gq}(z) && (8)\\
P_{\bar{q}\bar{q}}(z) &= P_{qq}(z) && (9)
\end{aligned}$$

The plus sign in the denominator simply subtracts the singular piece:

$$\frac{f(z)}{(1-z)_+} = \frac{f(z)-f(1)}{1-z}. \tag{10}$$