1. Using the Friedmann equation

\[ H^2 = \frac{8\pi}{3} G \left( \rho_m + \frac{\Lambda}{8\pi} \right) - \frac{k}{R^2}, \quad (1) \]

solve the evolution of the scale parameter \( R \) as a function of time with vanishing cosmological constant \( \Lambda = 0 \). Here, \( H = \dot{R}/R \), and the matter density \( \rho_m \) changes as \( R^{-3} \). Plot the evolution of the scale parameter \( R \) for closed \( k = 1 \), flat \( k = 0 \) and open \( k = -1 \) Universes.

2. We now discuss the case of flat Universe \((k = 0)\) with vanishing cosmological constant \( \Lambda = 0 \). Use the range of the Hubble constant from measurements \( H_0 = 50\text{--}80 \text{ km/sec/Mpc} \), plot the evolution of the scale parameter \( R \) and determine the range for the age of the Universe. Compare the result to the estimated 95% CL lower bound of 9.8 Gyr and the central value of 11.5 Gyr on the age of Universe (B. Chaboyer, P. Demarque, P. Kernan, and L. M. Krauss, Ap. J. 494, 96 (1998), astro-ph/9706128) and discuss what range of \( H_0 \) is consistent.

3. Study the flat Universe but now with cosmological constant, and draw a contour diagram of the age of the Universe on \((h, \Omega_m)\) plane, for \( 0 \leq h \leq 1 \) and \( 0 \leq \Omega_0 \leq 1 \). Here, \( h = H_0/(100 \text{ km/sec/Mpc}) \), and \( \Omega_m = \rho_m/\rho_c \) with \( \rho_c = 3H_0^2/8\pi G \). Note that the flatness \( k = 0 \) relates \( \Omega_m \) to the cosmological constant for fixed \( h \).

4. Repeat the same calculation for open Universe \( k = -1 \) without a cosmological constant, and draw a contour diagram of the age of the Universe on \((h, \Omega_m)\) plane.