229C HW # 5 (due Nov 12)

1. Using the Friedmann equation

$$H^2 = \frac{8\pi}{3} G \left(\rho_m + \frac{\Lambda}{8\pi} \right) - \frac{k}{R^2} \ , \qquad (1)$$

solve the evolution of the scale parameter R as a function of time with vanishing cosmological constant $\Lambda = 0$. Here, $H = \dot{R}/R$, and the matter density ρ_m changes as R^{-3}. Plot the evolution of the scale parameter R for closed $k = 1$, flat $k = 0$ and open $k = -1$ Universes.

2. We now discuss the case of flat Universe ($k = 0$) with vanishing cosmological constant $\Lambda = 0$. Use the range of the Hubble constant from measurements $H_0 = 50$–80 km/sec/Mpc, plot the evolution of the scale parameter R and determine the range for the age of the Universe. Compare the result to the estimated 95% CL lower bound of 9.8 Gyr and the central value of 11.5 Gyr on the age of Universe (B. Chaboyer, P. Demarque, P. Kernan, and L. M. Krauss, *Ap. J.* **494**, 96 (1998), astro-ph/9706128) and discuss what range of H_0 is consistent.

3. Study the flat Universe but now with cosmological constant, and draw a contour diagram of the age of the Universe on (h, Ω_m) plane, for $0 \leq h \leq 1$ and $0 \leq \Omega_0 \leq 1$. Here, $h = H_0/(100 \text{ km/sec/Mpc})$, and $\Omega_m = \rho_m/\rho_c$ with $\rho_c = 3H_0^2/8\pi G$. Note that the flatness $k = 0$ relates Ω_m to the cosmological constant for fixed h.

4. Repeat the same calculation for open Universe $k = -1$ *without* a cosmological constant, and draw a contour diagram of the age of the Universe on (h, Ω_m) plane.