229C HW # 6 (due Nov 25, 5pm)

1. Definition of the "luminosity distance" is $d_L^2 \equiv L/(4\pi F) = R_0^2 r^2 (1+z)^2$ with the comoving distance r. The propagation of light is determined by $ds^2 = 0$, or

$$\int \frac{dt}{R(t)} = \int \frac{r^2 dr}{\sqrt{1-kr^2}}, \tag{1}$$

which relates r to $(1+z) = R_0/R(t)$. Show that

$$H_0 d_L = \frac{1}{q_0^2}\left(z q_0 + (q_0 - 1)\left(\sqrt{2 q_0 z + 1} - 1\right)\right), \tag{2}$$

for a matter dominated open Universe with $q_0 = \Omega_m/2$ and $\Omega_\Lambda = 0$. Expand the above formula to find the approximation up to $O(z^3)$.

2. Using $T_\gamma = 2.728$ K for the cosmic microwave background radiation (CMBR), calculate the contribution of the CMBR to the current energy density Ω_γ.

3. Calculate the baryon-to-photon ratio $\eta = n_B/n_\gamma$ as a function of the current contribution of baryons to the comic energy density $\Omega_B = \rho_B/\rho_c$.

4. Start from the era $T > 1$ MeV when neutrinos were in thermal equilibrium with electrons, positrons and photons. At $T \sim 1$ MeV neutrinos decouple from the rest. At $T \ll m_e$, all electrons and positrons annihilate into photons except about 10^{-10} of the electrons that are "excess" over positrons. Using the conservation of entropy before and after the electron-positron annihilations, calculate the ratio of the neutrino temperature and the photon temperature. Calculate the current number density of the neutrinos using $T_\gamma = 2.728$ K. Also obtain the energy density if all three neutrinos have a common mass m_ν of 4 eV.