Physics 233B (Murayama)
HW #1, due Sep 24, 12:30 pm

1. The synchrotron radiation loss is given by

$$P = \frac{1}{6\pi\epsilon_0}\frac{e^2a^2\gamma^4}{c^3}, \qquad a = \frac{v^2}{R}, \tag{1}$$

where R is the radius of curvature. The formula for the instantaneous luminosity is

$$\mathcal{L} = f_c\frac{N_+N_-}{4\pi\sigma_x^*\sigma_y^*}S \tag{2}$$

with $S \approx 1$. Determine what circumference is required for a 2 TeV e^+e^- machine to maintain the same event rate for annihilation processes (assuming $1/s$ scaling) and power consumption to compensate for the synchrotron radiation loss as LEP-II. Keep the machine parameters $\sigma_{x,y}^*$, S fixed.

2. The lowest order QCD predictions (not a very good approximation!) are (see, *e.g.*, P. B. Mackenzie and G. P. Lepage, "QCD Corrections To The Gluonic Width Of The Υ Meson," Phys. Rev. Lett. **47**, 1244 (1981).)

$$\Gamma(J/\psi \to ggg) = \frac{160}{81}(\pi^2-9)\alpha_s^3(M)\frac{|\psi_{NR}(0)|^2}{M^2}\ , \tag{3}$$

$$\Gamma(J/\psi \to \mu^+\mu^-) = 16\pi Q_c^2\alpha^2\frac{|\psi_{NR}(0)|^2}{M^2}\ , \tag{4}$$

$$\Gamma(J/\psi \to q\bar{q}) = 16\pi Q_c^2N_cQ_q^2\alpha^2\frac{|\psi_{NR}(0)|^2}{M^2}\ . \tag{5}$$

Here, $Q_c = \frac{2}{3}$ is the charge of the charm quark. Use information from the particle listsings on J/ψ to estimate $\alpha_s(M)$ at $M = 3097$ MeV.

3. Work out the $e^+e^- \to$ hadrons in the vicinity of J/ψ including the intereference effects and compare it with the data. No Feynman diagram calculations! Just rescale from $\sigma_{pt} = \frac{86.6\ \text{nb}}{s/\text{GeV}^2}$ using the width and branching fractions from the PDG. Make sure to separate ggg final state (no intereference) and γ^* decay (with interference). Fold together with energy resultion to compare with data in V. M. Aulchenko *et al.* [KEDR Collaboration], "New precision measurement of the J/ψ and ψ' meson masses," Phys. Lett. B **573**, 63 (2003), Fig. 7.